Journa of Differential Geometry, Applications and Aspects **Vol. 1** *1 (2024): 34-48*

USING LADM FOR SOLVING THE DYNAMIC SYSTEM GOVERNING COVID-19 OUTBREAK

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Abstract.

An outbreak of the 2019 novel corona-virus disease (COVID-19) in Wuhan, China has spread quickly in all the world. The pivotal aim of the present work is to achieve an analytic approximate solution for the fractional order outbreak model of COVID-19 utilizing LADM. First the transmissibility of corona-virus is introduced as a nonlinear fractional order dynamic system with six classes of population, then by employing LADM, the approximated solutions in the series form are obtained. Further, novel simulations for all cases of results are provided to validate the applicability and effectiveness of the propounded scheme. Also we analyze the effect of some important parameters of the model on the control of the virus spread. The outcomes of this research reveal that the LADM is computationally very effective to analysis nonlinear fractional differential equations arises in daily life problems.

1. Introduction

At the end of 2019, the world was effected by a novel infectious disease caused by a new kind of coronaviruses, namely Covid-19. There are four kind of human coronaviruses, α, β, γ and δ or SARS-CoV. But the discovered coronavirus in Wuhan city of China at December 2019, was different from other kinds. As reported by the World Health Organization (WHO) [1], this virus is thought to be transmitted from animals to humans and authors of [2], introduced the bats as the probably origin of this virus. Theory of transmission of the virus form bats to the hosts (it may be the wild animals) and the hunting of the hosts and their travel to the seafood market (Huanan Seafood Wholesale Market), was expressed in many literature, such as [3]-[4]. Unfortunately, the spread of this virus was too rapid in China and in spite of the quarantine restrictions of Chinese government for Whuan city of Hubei province it transformed to global epidemic at the beginning of 2020.
Historical evidence show the negative influence of the countries by the infectious pandemics in many fields, such as health and finance. Unfortunately after about 13 months from beginning of this pandemic, the definitive treatment for it has not been found yet. Therefore the only way for controlling the spread of coronavirus is understanding and mathematical modelling of the dynamic of its transmission.
In this regard some researchers have studied the dynamic of the infectious diseases. In [5] the temporal dynamics of the coronavirus disease 2019 outbreak in China, Italy and France in special time interval was analyzed and the mathematical model of outbreak was considered as a system of nonlinear differential equations. Atangana in [6] suggested a mathematical model taking into account the possibility of transmission

2010 *Mathematics Subject Classification.* 58E11, 53C50, 53B30.

Key words and phrases. Laplace transform, Adomian decomposition method, COVID-19 outbreak, Caputo fractional derivative.

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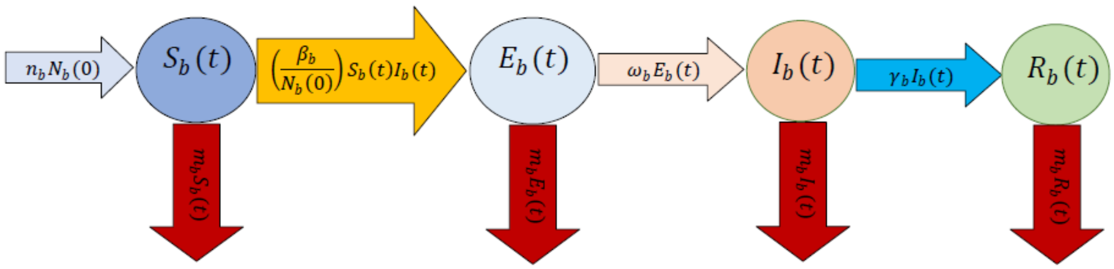

FIGURE 1. Diagram of COVID-19 spread in the bats.

of COVID-19 from dead bodies to humans and the effect of lock-down by studying three cases. Also he introduced the novel differential and integral operators to the suggested model. In [7] authors are described the brief details of interaction among the bats and unknown hosts, then among the peoples and the infections reservoir (seafood market) and formulated the model of spread as nonlinear system of fractional differential equations.

In this study the mathematical model of spread of coronavirus based on statistical data of Wuhan city, is considered as nonlinear system of fractional differential equations where fractional derivative is defined in Caputo sense. The unknowns of system are classified in six groups: the susceptible, exposed, symptomatic infected, asymptotically infected, recovered and dead peoples. In solving the purposed model for different values of fractional derivative order, first Laplace transform and Adomian decomposition method are applied for introducing the solutions as infinite time series. Then utilizing the iterated Shanks transform, an accelerating convergence technique, on the truncated solution series, we obtain approximate solutions of system. In the following, for analysis the effect of shedding rate from infected cases to the susceptible people and the reservoir, also the shedding rate from the reservoir to the susceptible people, we solved the non-fractional model for different values of model parameters.

The organization of this paper is as follows. Mathematical modelings of transmission in animal and human societies, based on Caputo fractional derivative are described in section 2. Section 3 is devoted to introducing the suggested approaches for solving the relative human society model, and the purposed approaches are utilized for analysis a case study 4. Conclusion of this paper is given in section 5, where the impact of principal parameters of main problem and inducing the fractional order are discussed in details.

2. Mathematical modelling of transmission

2.1. **Model formulation for bats and hosts.** Most researchers have the same theory about the primary transmission of the coronavirus; the believe that this virus belongs to the bats population and the initial spread occurred between the bats and then transmitted to unknown hosts (probably some wild animals). By hunting the hosts and sending them to food markets (almost seafood markets), the fundamental spread in human population was happened. The seafood markets are considered as the reservoir in this study. Figure 1 is useful in schematic understanding of the transmission.

For modelling the dynamic of outbreak between the bats and the hosts, the bats and the hosts are classified in four group. The susceptible, the exposed, the infected and the recovered or removed groups. Figures 1-2 show the diagrams of spread of COVID-19 in the bats and hosts populations, respectively. It should be noticed that the number of exposed hosts depend to the number of infected bats and this is shown in figure 2. The spread dynamic in bats and hosts is defined as [7]-[8]

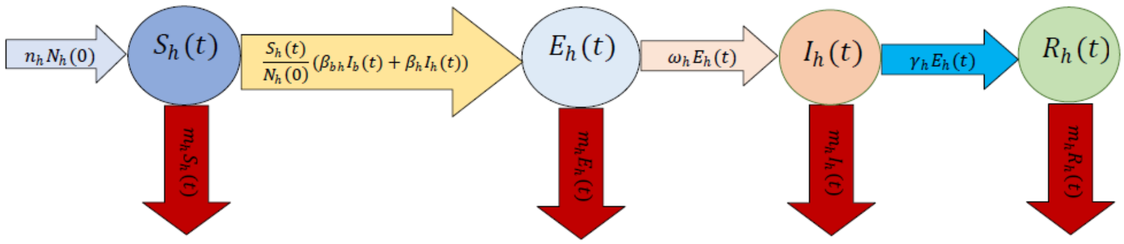

FIGURE 2. Diagram of COVID-19 spread in the hosts.

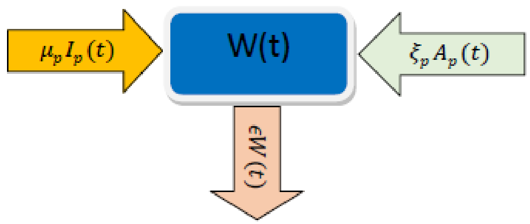

FIGURE 3. Diagram of the changing in the reservoir.

$$\begin{cases} \frac{dS_b}{dt} = n_b N_b(0) - m_b S_b - \frac{\beta_b}{N_b(0)} S_b I_b, \\ \frac{dE_b}{dt} = \frac{\beta_b}{N_b(0)} S_b I_b - (\omega_b + m_b) E_b, \\ \frac{dI_b}{dt} = \omega_b E_b - (\gamma_b + m_b) I_b, \\ \frac{dR_b}{dt} = \gamma_b I_b - m_b R_b, \\ \frac{dS_h}{dt} = n_h N_h(0) - m_h S_h - \frac{\beta_{bh}}{N_h(0)} S_h I_b - \frac{\beta_h}{N_h(0)} S_h I_h, \\ \frac{dE_h}{dt} = \frac{\beta_{bh}}{N_h(0)} S_h I_h + \frac{\beta_h}{N_h(0)} S_h I_h - (\omega_h + m_h) E_h, \\ \frac{dI_h}{dt} = \omega_h E_h - (\gamma_h + m_h) I_h, \\ \frac{dR_h}{dt} = \gamma_h I_h - m_h R_h, \end{cases} \tag{2.1}$$

subject to the following non-negative initial conditions

$$S_b(0) = s_{b0}, \quad E_b(0) = e_{b0}, \quad I_b(0) = i_{b0}, \quad R_b(0) = r_{b0},$$

$$S_h(0) = s_{h0}, \quad E_h(0) = e_{h0}, \quad I_h(0) = i_{h0}, \quad R_h(0) = r_{h0}.$$

where the coefficients and unknowns of equation (2.1) for the hosts population are defined in table 1. In the table 1, $N_h(0)$ is the total number of hosts and we have $N_h(0) = S_h(0) + E_h(0) + I_h(0) + R_h(0)$. It is clear that the unknowns and the parameters for the bats population can be defined similarly, without β_{bh}.

TABLE 1. descriptions of some parameters and unknowns of equation (2.1)

unknowns	description
S_h	Susceptible hosts
E_h	Exposed hosts
I_h	Symptomatic infected hosts
R_h	Recovered and dead hosts
S_b	Susceptible bats
E_b	Exposed bats
I_b	Symptomatic infected bats
R_b	Recovered and dead bats
parameters	description
n_h (n_b)	Birth rate of hosts (bats)
m_h (m_b)	Mortality rate of hosts (bats)
β_h (β_b)	Transmission rate from I_h to S_h (I_b to S_b)
β_{bh}	Transmission rate from I_b to S_h
ω_h (ω_b)	Incubation period in the hosts (bats) body
γ_h (γ_b)	Recovery or removal rate of I_h (I_b)

2.2. **Model formulation for people.** Motivated by the transmission dynamic for animals, the mathematical model for the dynamic of the transmissibility of the coronavirus in people is derived in [7] as follows:

$$\begin{cases} \frac{dS_p}{dt} = m_P N_p(0) - m_p S_p - \frac{\beta_p}{N_p} S_p \left(I_p + kA_p\right) - \beta_W S_p W, \\ \frac{dE_p}{dt} = \frac{\beta_p}{N_p} S_p \left(I_p + kA_p\right) + \beta_W S_p W - \left[(1-\delta_p)\,\omega_p + \delta_p \rho_p + m_p\right] E_p, \\ \frac{dI_p}{dt} = (1-\delta_p)\,\omega_p E_p - (\gamma_p + m_p)\, I_p, \\ \frac{dA_p}{dt} = \delta_p \rho_p E_p - (\alpha_p + m_p)\, A_p, \\ \frac{dR_p}{dt} = \gamma_p I_p + \alpha_p A_p - m_p R_p, \\ \frac{dW}{dt} = \mu_p I_p + \xi_p A_p - \varepsilon W, \end{cases} \tag{2.2}$$

subject to the following non-negative initial conditions

$$S_p(0) = S_0, \qquad E_p(0) = E_0, \qquad I_p(0) = I_0,$$

$$A_p(0) = A_0, \qquad R_p(0) = R_0, \qquad W(0) = W_0,$$

where the definitions of S_p, E_p, I_p and R_p in system (2.2) are similarly to the ones in system (2.1), and A_p and W show the asymptomatic infected people and the COVID-19 in reservoir in time t. Even though W is one of the unknowns of system (2.2), but it does not belong to the human chain of transmission, so the its change process diagram was drawn separately in figure 3. Figure 4 shows the diagram of spread in human society and the description of parameters of system (2.2) are given in table 2. The main aim of this paper

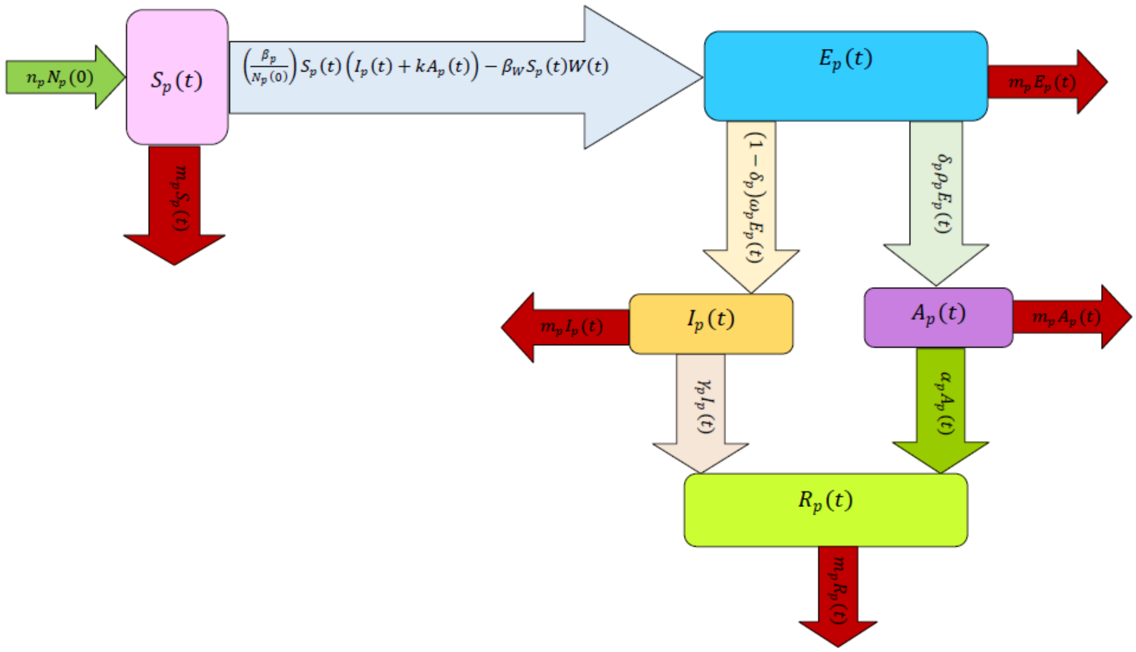

FIGURE 4. Diagram of COVID-19 spread in the human society.

TABLE 2. model parameters and their descriptions

variables	description
$N_p(0)$	The total number of people
m_p	Natural mortality rate
ω_p	Incubation period
ρ_p	Latent period
γ_p	Removal or recovery rate of I_p
α_p	Removal or recovery rate of A_p
μ_p	Shedding coefficients from I_p to W
ξ_p	Shedding coefficients from A_p to W
δ_p	Proportion of asymptomatic infection
β_p	Transmission rate from I_p to S_p
β_W	Transmission rate from W to S_p
k	Transmissibility multiple
ε	Removing rate of virus from W

is solving the Caputo fractional form of system (2.2) as

$$\begin{cases} {}_0^{\mathbf{c}}D_t^{\alpha} S_p = m_P N_p(0) - m_p S_p - \frac{\beta_p}{N_p} S_p\left(I_p + kA_p\right) - \beta_W S_p W, \\ {}_0^{\mathbf{c}}D_t^{\alpha} E_p = \frac{\beta_p}{N_p} S_p\left(I_p + kA_p\right) + \beta_W S_p(t) W - \left[(1-\delta_p)\,\omega_p + \delta_p\rho_p + m_p\right] E_p(t), \\ {}_0^{\mathbf{c}}D_t^{\alpha} I_p = (1-\delta_p)\,\omega_p E_p - (\gamma_p + m_p)\, I_p, \\ {}_0^{\mathbf{c}}D_t^{\alpha} A_p = \delta_p\rho_p E_p - (\alpha_p + m_p)\, A_p, \\ {}_0^{\mathbf{c}}D_t^{\alpha} R_p = \gamma_p I_p + \alpha_p A_p - m_p R_p, \\ {}_0^{\mathbf{c}}D_t^{\alpha} W(t) = \mu_p I_p + \xi_p A_p - \varepsilon W, \end{cases} \tag{2.3}$$

where ${}_0^{\mathrm{c}}D^{\alpha}$ is the fractional differential of order α, for finding the unknowns S_p, I_p, E_p, A_p, R_p and W, where $0 < \alpha \leq 1$. For this purpose first by using Laplace transform the system of Caputo fractional differential equations (2.3) is reduced to a system of algebraic equations and then Adomian decomposition method is applied for obtaining the solutions.
Now we briefly present the known definition of Caputo fractional derivative.

Definition 2.1. The Caputo fractional derivative operator of order nonnegative α is defined as [9]

$$D^{\alpha} f(x) = \frac{1}{\Gamma(n-\alpha)} \int_0^x \frac{f^{(n)}(t)}{(x-t)^{\alpha+1-n}} dt, \quad n-1 < \alpha \leq n, \quad n \in N. \tag{2.4}$$

Based on applying the Laplace transform on the system (2.3), we express the following property for Caputo derivative.
The Laplace transform of Caputo fractional derivative is as follows

$$\mathcal{L}\left\{D^{\alpha} f\right\} = s^{\alpha} \mathcal{L}\{f\} - \sum_{k=0}^{n-1} s^{\alpha-k-1} f^{(k)}(0), \qquad n-1 < \alpha \leq n. \tag{2.5}$$

3. Numerical implementation

In this section Laplace transform via Adomian decomposition method are used for solving nonlinear system (2.3). First by applying Laplace transform to both sides of the model (2.3) we get

$$\begin{aligned}
\mathcal{L}\left\{S_p\right\} &= \tfrac{S_0}{s} + \tfrac{n_P N_p(0)}{s^{\alpha+1}} - \tfrac{m_p}{s^{\alpha}} \mathcal{L}\left\{S_p\right\} - \tfrac{\beta_p}{s^{\alpha} N_p} \mathcal{L}\left\{S_p\left(I_p + k A_p\right)\right\} - \tfrac{\beta_W}{s^{\alpha}} \mathcal{L}\left\{S_p W\right\}, \\
\mathcal{L}\left\{E_p\right\} &= \tfrac{E_0}{s} + \tfrac{\beta_p}{s^{\alpha} N_p} \mathcal{L}\left\{S_p\left(I_p + k A_p\right)\right\} + \tfrac{\beta_W}{s^{\alpha}} \mathcal{L}\left\{S_p W\right\} - \tfrac{(1-\delta_p)\omega_p + \delta_p \rho_p + m_p}{s^{\alpha}} \mathcal{L}\left\{E_p\right\}, \\
\mathcal{L}\left\{I_p\right\} &= \tfrac{I_0}{s} + \tfrac{(1-\delta_p)\omega_p}{s^{\alpha}} \mathcal{L}\left\{E_p\right\} - \tfrac{\gamma_p + m_p}{s^{\alpha}} \mathcal{L}\left\{I_p\right\}, \\
\mathcal{L}\left\{A_p\right\} &= \tfrac{A_0}{s} + \tfrac{\delta_p \rho_p}{s^{\alpha}} \mathcal{L}\left\{E_p\right\} - \tfrac{\alpha_p + m_p}{s^{\alpha}} \mathcal{L}\left\{A_p\right\}, \\
\mathcal{L}\left\{R_p\right\} &= \tfrac{R_0}{s} + \tfrac{\gamma_p}{s^{\alpha}} \mathcal{L}\left\{I_p\right\} + \tfrac{\alpha_p}{s^{\alpha}} \mathcal{L}\left\{A_p\right\} - \tfrac{m_p}{s^{\alpha}} \mathcal{L}\left\{R_p\right\}, \\
\mathcal{L}\left\{W\right\} &= \tfrac{W_0}{s} + \tfrac{\mu_p}{s^{\alpha}} \mathcal{L}\left\{I_p\right\} + \tfrac{\xi_p}{s^{\alpha}} \mathcal{L}\left\{A_p\right\} - \tfrac{\varepsilon}{s^{\alpha}} \mathcal{L}\left\{W\right\}.
\end{aligned} \tag{3.1}$$

Now we write the unknowns of the system (3.1) in infinite series as

$$S_p(t) = \sum_{j=0}^{\infty} s_j(t), \qquad E_p(t) = \sum_{j=0}^{\infty} e_j(t), \qquad I_p(t) = \sum_{j=0}^{\infty} i_j(t),$$

$$A_p(t) = \sum_{j=0}^{\infty} a_j(t), \qquad R_p(t) = \sum_{j=0}^{\infty} r_j(t), \qquad W(t) = \sum_{j=0}^{\infty} w_j(t). \tag{3.2}$$

Also the nonlinear variable terms of the system (3.1) are written by Adomian polynomials as

$$S_p(t) A_p(t) = \sum_{j=0}^{\infty} \mathcal{A}_j(t) = \sum_{j=0}^{\infty} \frac{1}{\Gamma(j+1)} \left[\frac{d^j}{d\lambda^j}\left(\sum_{l=0}^{j} \lambda^l s_l(t) \sum_{l=0}^{j} \lambda^l a_l(t)\right)\right]_{\lambda=0},$$

$$S_p(t)I_p(t)=\sum_{j=0}^{\infty}\mathcal{B}_j(t)=\sum_{j=0}^{\infty}\frac{1}{\Gamma(j+1)}\left[\frac{d^j}{d\lambda^j}\left(\sum_{l=0}^{j}\lambda^l s_l(t)\sum_{l=0}^{j}\lambda^l i_l(t)\right)\right]_{\lambda=0}, \tag{3.3}$$

$$S_p(t)W(t)=\sum_{j=0}^{\infty}\mathcal{C}_j(t)=\sum_{j=0}^{\infty}\frac{1}{\Gamma(j+1)}\left[\frac{d^j}{d\lambda^j}\left(\sum_{l=0}^{j}\lambda^l s_l(t)\sum_{l=0}^{j}\lambda^l w_l(t)\right)\right]_{\lambda=0}.$$

Now by substituting equations (3.2)-(3.3) in (3.1) and utilizing Adomian decomposition method we get the Laplace transform of initial conditions as

$$\mathcal{L}\{s_0\}=\frac{S_0}{s}+\frac{n_P N_p(0)}{s^{\alpha+1}},\qquad \mathcal{L}\{e_0\}=\frac{E_0}{s},\qquad \mathcal{L}\{i_0\}=\frac{I_0}{s},$$

$$\mathcal{L}\{a_0\}=\frac{A_0}{s},\qquad \mathcal{L}\{r_0\}=\frac{R_0}{s},\qquad \mathcal{L}\{w_0\}=\frac{W_0}{s},$$

and consequently the following recursive relations

$$\begin{aligned}
\mathcal{L}\{s_{j+1}\}&=-\tfrac{m_p}{s^\alpha}\mathcal{L}\{s_j\}-\tfrac{\beta_p}{s^\alpha N_p}\Big(\mathcal{L}\{\mathcal{B}_j\}+k\mathcal{L}\{\mathcal{A}_j\}\Big)-\tfrac{\beta_W}{s^\alpha}\mathcal{L}\{\mathcal{C}_j\},\\
\mathcal{L}\{e_{j+1}\}&=\tfrac{\beta_p}{s^\alpha N_p}\Big(\mathcal{L}\{\mathcal{B}_j\}+k\mathcal{L}\{\mathcal{A}_j\}\Big)+\tfrac{\beta_W}{s^\alpha}\mathcal{L}\{\mathcal{C}_j\}-\tfrac{(1-\delta_p)\omega_p+\delta_p\rho_p+m_p}{s^\alpha}\mathcal{L}\{e_j\},\\
\mathcal{L}\{i_{j+1}\}&=\tfrac{(1-\delta_p)\omega_p}{s^\alpha}\mathcal{L}\{e_j\}-\tfrac{\gamma_p+m_p}{s^\alpha}\mathcal{L}\{i_j\},\\
\mathcal{L}\{a_{j+1}\}&=\tfrac{\delta_p\rho_p}{s^\alpha}\mathcal{L}\{e_j\}-\tfrac{\alpha_p+m_p}{s^\alpha}\mathcal{L}\{a_j\},\\
\mathcal{L}\{r_{j+1}\}&=\tfrac{\gamma_p}{s^\alpha}\mathcal{L}\{i_j\}+\tfrac{\alpha_p}{s^\alpha}\mathcal{L}\{a_j\}-\tfrac{m_p}{s^\alpha}\mathcal{L}\{r_j\},\\
\mathcal{L}\{w_{j+1}\}&=\tfrac{\mu_p}{s^\alpha}\mathcal{L}\{i_j\}+\tfrac{\xi_p}{s^\alpha}\mathcal{L}\{a_j\}-\tfrac{\varepsilon}{s^\alpha}\mathcal{L}\{w_j\}.
\end{aligned} \tag{3.4}$$

By taking inverse Laplace transform of linear system (3.4), we obtain the analytical solutions of nonlinear system (2.1) as (3.2). The M-term approximation for analytical solutions of the Laplace Adomian decomposition method are defined as

$$s_{p,M}(t)=\sum_{j=0}^{M-1}s_j,\qquad e_{p,M}(t)=\sum_{j=0}^{M-1}e_j,\qquad i_{p,M}(t)=\sum_{j=0}^{M-1}i_j,$$

$$a_{p,M}(t)=\sum_{j=0}^{M-1}a_j,\quad r_{p,M}(t)=\sum_{j=0}^{M-1}r_j,\quad w_M(t)=\sum_{j=0}^{M-1}w_j. \tag{3.5}$$

In the following, iterated Shanks transform is applied to accelerate the convergence of approximate solutions (3.5).

3.1. **Iterated Shanks transform.** Assume that the solutions $s_{p,i}$ are computed for $i=1,2,3,...,M$, we define $s_{p,i}=\mathcal{S}[s]_i^0$. Now for $j\geq 1$, define

$$\mathcal{S}[s]_i^j=\frac{\mathcal{S}[s]_i^{j-1}\mathcal{S}[s]_{i+2}^{j-1}-\left(\mathcal{S}[s]_{i+1}^{j-1}\right)^2}{\mathcal{S}[s]_i^{j-1}-2\mathcal{S}[s]_{i+1}^{j-1}+\mathcal{S}[s]_{i+2}^{j-1}}. \tag{3.6}$$

TABLE 3. Estimated parameters of model (2.3) for Wuhan at 21 to January 28, 2020

variables	description
m_p	0.000035678 [10]
ω_p	0.00047876
ρ_p	0.005
γ_p	0.09871
α_p	0.854302
μ_p	0.000398
ξ_p	0.001
δ_p	0.1243
β_p	0.05
β_W	0.000001231
k	0.02
ε	0.01

The result of iterated Shanks transform is defined as

$$\mathcal{S}\{s_{p,1},\ldots,s_{p,M}\} := \begin{cases} \mathcal{S}[s]_1^{\frac{M-1}{2}} & M \text{ is odd number}, \\ \frac{\mathcal{S}[s]_1^{\frac{M-2}{2}}+\mathcal{S}[s]_2^{\frac{M-2}{2}}}{2}, & M \text{ is even number}. \end{cases}$$

Similarly, we do this process for other approximate solutions e_p, i_p, a_p, r_p and w, respectively.

4. CASE STUDY: WHUAN CITY

In this section we report approximate solutions of system (2.3), obtained by aforementioned method, where the parameters and initial conditions of problem are estimated from clinical and hospital statistics of Wuhan city, where the initial population was $N_p(0) = 8,266,000$ [10]. The initial numbers of the six people groups of system (2.3) are reported as: $E_0 = 200,000$, $I_0 = 282$, $A_0 = 200$, $W(0) = 50,000$ and $S_0 = 8,065,518$, it be considered that the number of the dead and the recovered people is zero, $R_0 = 0$. Also the parameters of system (2.3) are calculated by curve fitting the introduced mathematical model and the real data. The estimated parameters for this case study are given in table 3. The fractional model (2.3) is solved by introduced method for different values of α and the obtained series solutions are truncated for $M = 5$. Then iterated Shanks transform $\mathcal{S}[.]_1^2$, is employed for increasing the convergence of numerical solutions. The results are shown in figures 5-10. It can be easily seen in this figures that the approximated numbers are in good agreement with the results of [7], which approximated the numbers for 120 days.
Now in order to analysis the effectness of shedding parameters μ_p, ξ_p and β_p, we solve the nonlinear model (2.3) for $\alpha = 1$ and different parameters μ_p, ξ_p and β_p. In this study these parameters based on our case study, Wuhan city, have taken as; $\mu_p = 0.000398$, $\xi_p = 0.001$ and $\beta_p = 0.05$.

By the obtained diagrams 5-10 of the approximated solutions, it can be seen clearly that the number of susceptible people will be decreased over time, while the number of infected people will be increased. This is admissible because the communications between the people of involved society by Covid-19, will increase the number of exposed people. It should be noticed that this virus has considerable large reproduction number, R_0. In some countries reproduction number was estimated of 3.58, that is, a single infected person can transmit this virus to 3.58 susceptible people. Another important factor in the dynamic of Covid-19 spread is its long incubation and latent period. In some cases the infected people have no symptom in two weeks and in this period they can transmit the virus to the susceptible people. On the other hand the number of

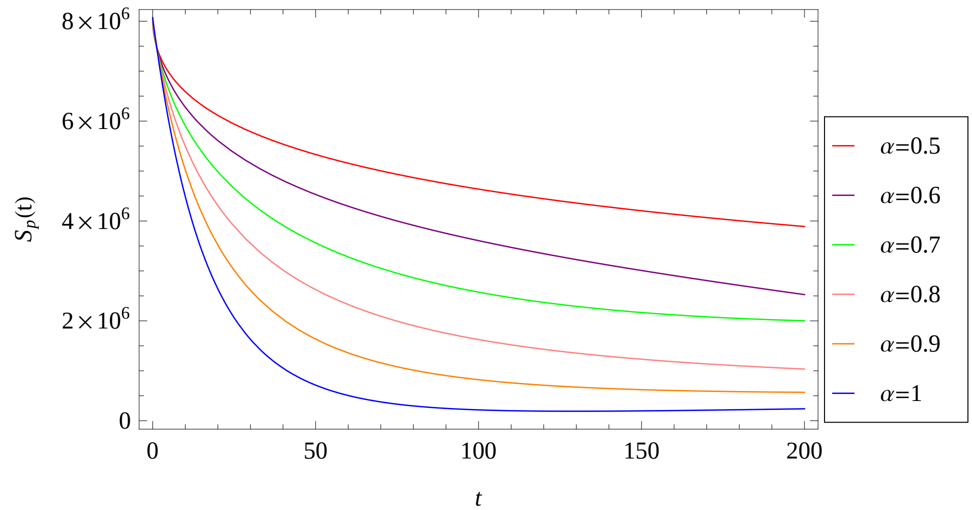

FIGURE 5. Approximated number of the suspectable people for some values of α in 200 days.

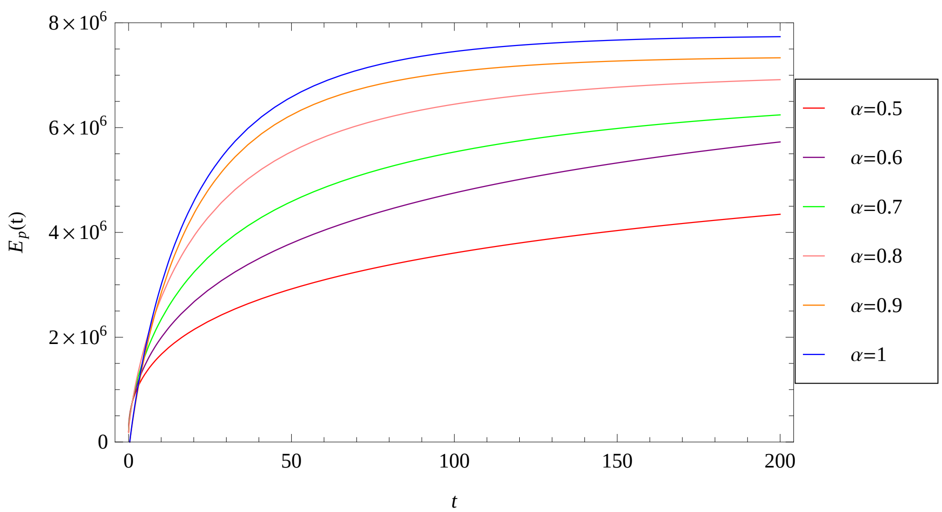

FIGURE 6. Approximated number of the exposed people for some values of α in 200 days.

virus at the reservoir will decrease over time, because by shut downing the food markets (especially seafood markets) and in general crowded centers, quarantining the infected and susceptible people, disinfecting the places and observing health tips, the shedding rate of virus will be vanished, approximately.

4.1. **Effect of μ_p.** For study the impact of shedding the corona virus from symptomatic infected people to the reservoir, on the solutions of system (2.3), we solved the non-fractional model ($\alpha = 1$) for some different μ_p. The results for $\mu_p = 0.04$, 0.000398, 0.000004 are shown in figure 11. Where $\mu_p = 0.000398$ is the value of the related shedding parameter for Wuhan and we assume two increased and decreased values for it. As we can see in the solution plots, by decreasing the shedding rate from the infected people to the reservoir, the number of the infected both symptomatic and asymptomatic and exposed people will be decreased. There is almost a unique theory about the initial reservoir of Covid-19 in Wuhan city, the seafood market

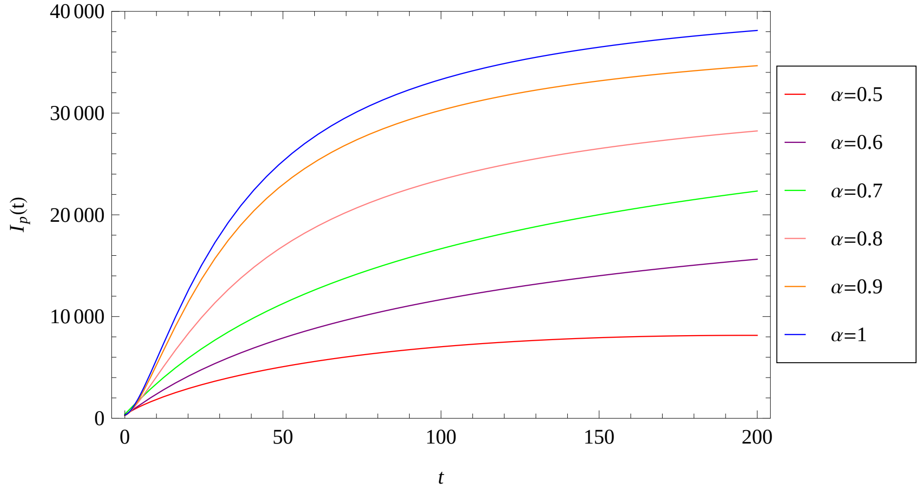

FIGURE 7. Approximated number of the symptomatic infected people for some values of α in 200 days.

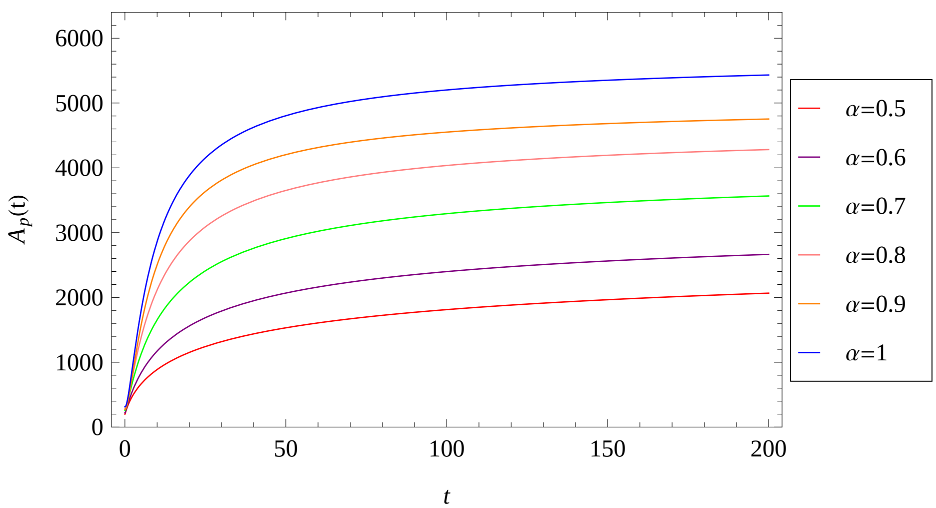

FIGURE 8. Approximated number of the asymptomatic infected people for some values of α in 200 days.

of Wuhan. But it is clear that the reservoirs of Corona virus in various countries are numerous, such as airports, universities, hyper markets, banks and etc . So some alternatives to reduce the shedding rate from the infected people into these places are shutting down the reservoirs and quarantine the infected people, both symptomatic and asymptomatic. By the plot of $R_p(t)$ in figure 11, it is clear that the number of recovered people is not affected by changing the value of μ_p.

4.2. **Effect of ξ_p.** In the following for understanding the affect of shedding parameter from asymptomatic infected people to reservoir, we perturbate the value of ξ_p in non-fractional model of system (2.1). We put $\xi_p = 0.1,\ 0.001, 0.00001$ and numerical results are shown in figure 12. As we can see in the plot $W(t)$ of figure 12, for $\xi_p = 0.00001$, after 107 days, the number of COVID-19 viruses in the reservoir approximately

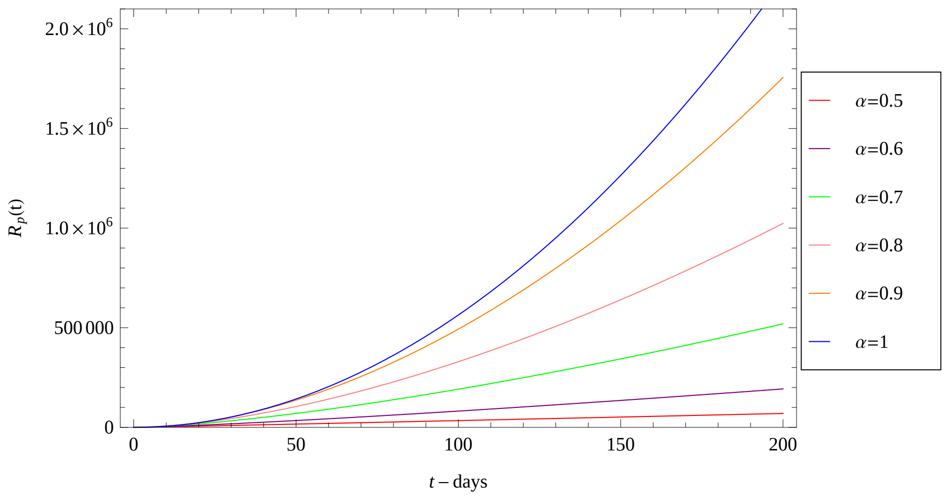

FIGURE 9. Approximated number of the recovered or dead people for some values of α in 200 days.

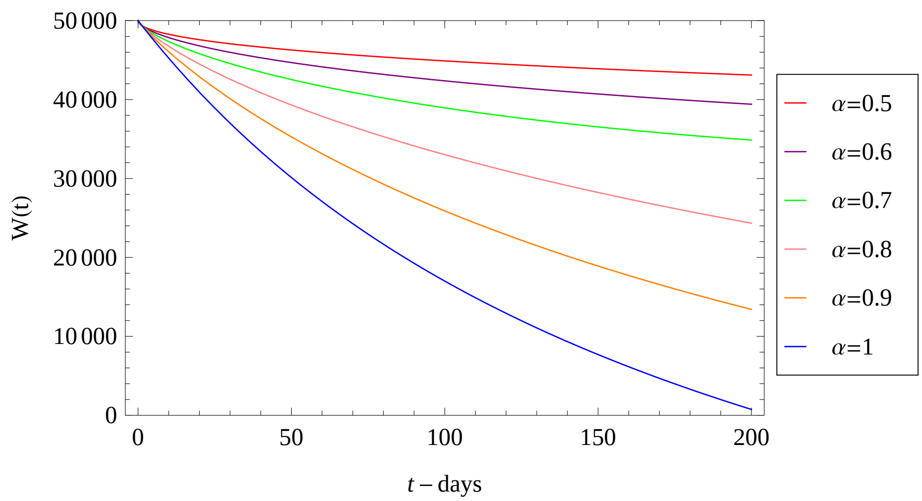

FIGURE 10. Approximated number of the recovered or dead people for some values of α in 200 days.

vanishes, that is closing or even limiting the reservoir activities for about 100 days has an important role in the control of the coronavirus spread.

4.3. **Effect of β_p.** One of the effective parameter in the control of spread COVID-19 is lack of communication between the infected and the susceptible people. In this section we solve the no-fractional system (2.2) for some different values of the related parameter, β_p. We choice two nearby values to main parameter, $\beta_p = 0.005,\ 0.5$. The plots of numerical results are shown in figure 13. As expected, by increasing the parameter β_p, the number of infected people will increase, to be more precise if the transmission rate from infected people to susceptible people increases ten times, then after 100 days the number of symptomatic infected people increases 1.21 times and the number of asymptomatic people increases 1.15 times. On the other hand by decreasing the transmission rate between I_p and S_p, after 100 days

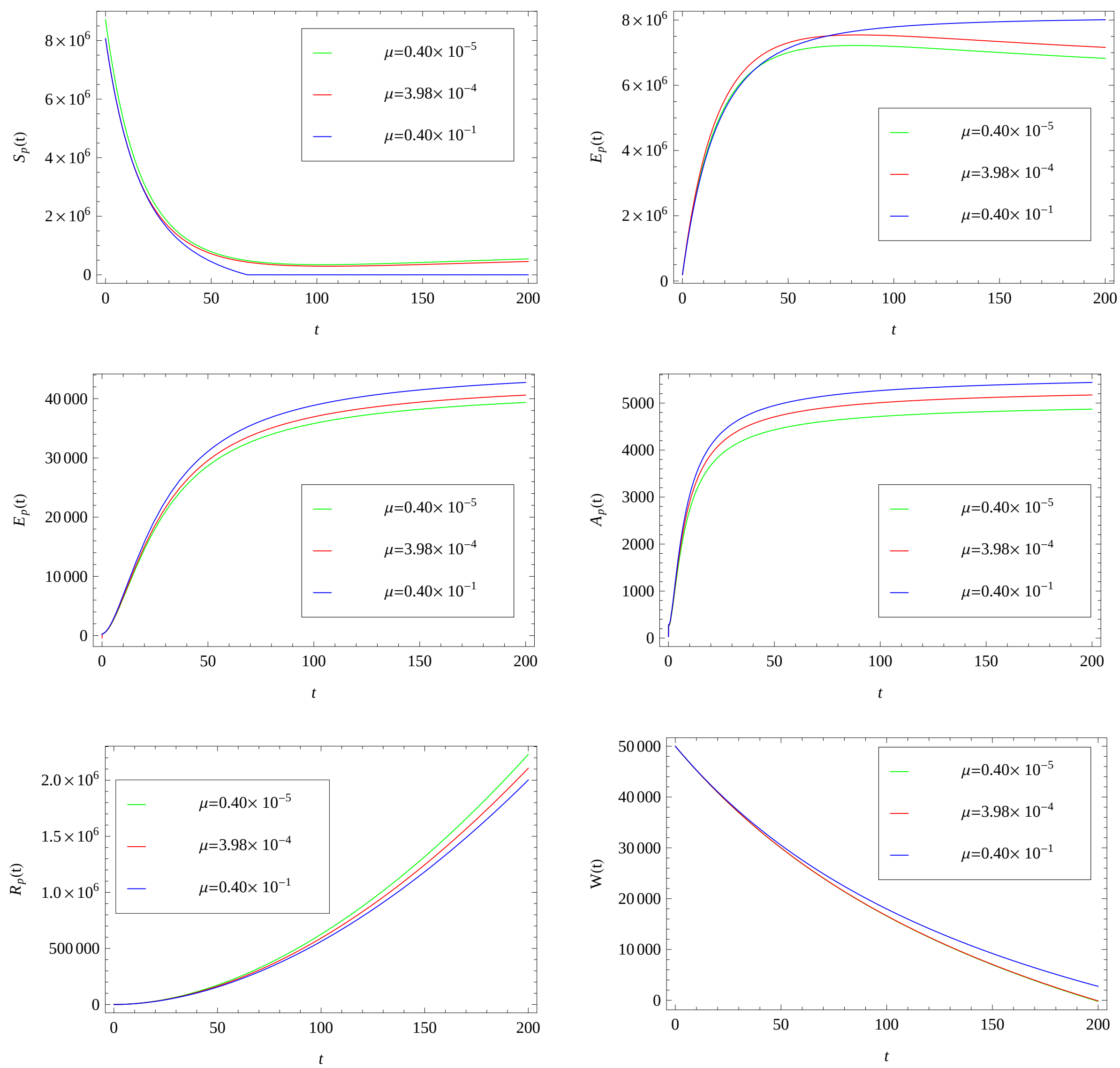

FIGURE 11. Approximated results for S_p, E_p, I_p, A_p, R_p and W for some different values of μ_p in 200-days.

5. Conclusion

In this research the mathematical modeling of Covid-19 virus spread is considered as a system of fractional differential equations in the Caputo sense. By using Laplace transform and Adomian decomposition method, the nonlinear fractional differential equations system reduced to some easily solvable recursive relations. Also in order to achieving fast convergence of numerical solutions, we employed iterative Shanks transform. For showing the efficiency of the presented method, the dynamic of Covid-19 in Wuhan city is studied and related model is solved by introduced approach. As we can see in figures 5-10, that fractional order of Covid-19 outbreak has more degree of freedom and therefore can be varied to get various responses of the different compartments of the proposed model. In the following the affect of some human parameters of

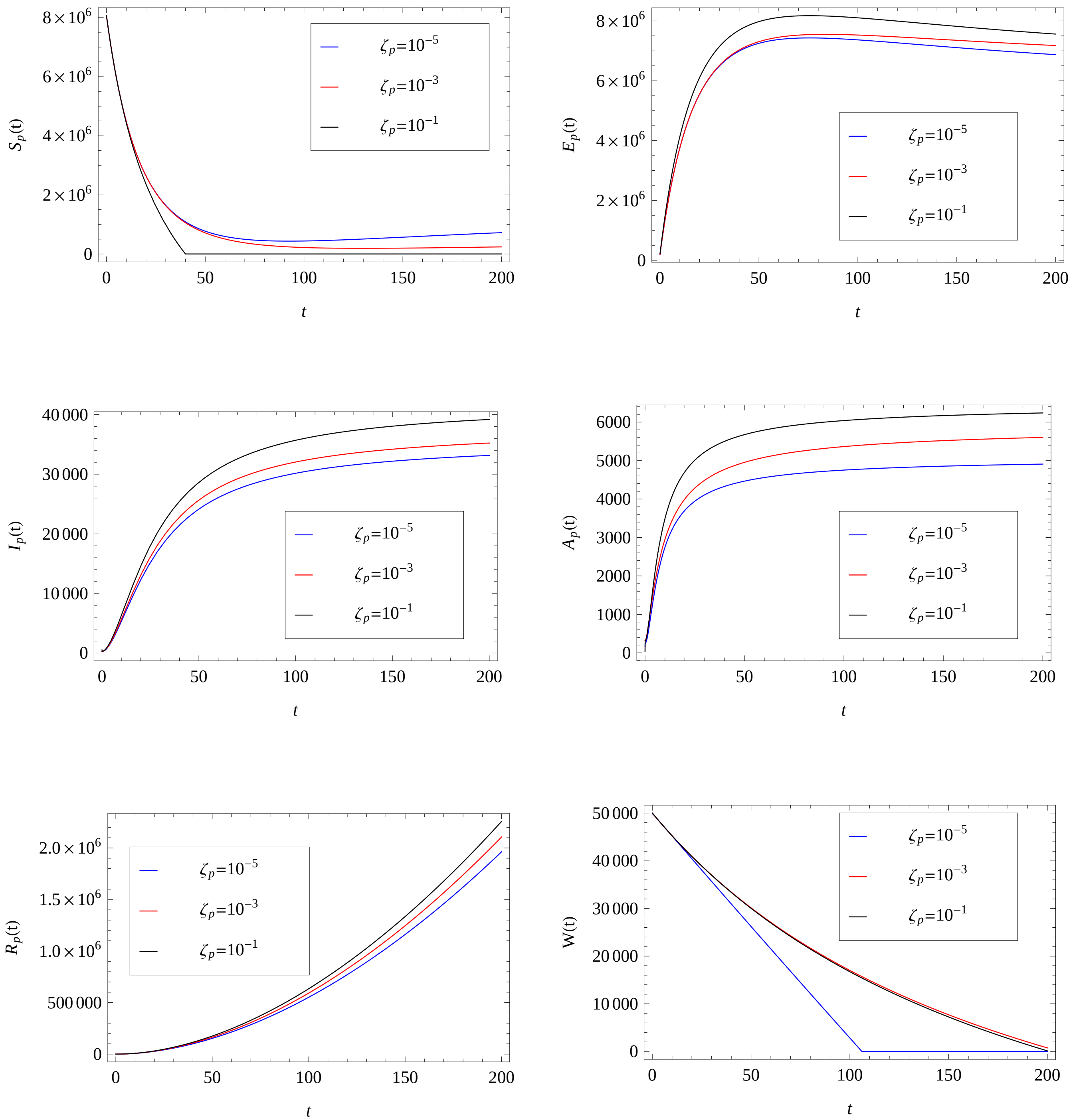

FIGURE 12. Approximated results for S_p, E_p, I_p, A_p, R_p and W for some different values of ξ_p in 200-days.

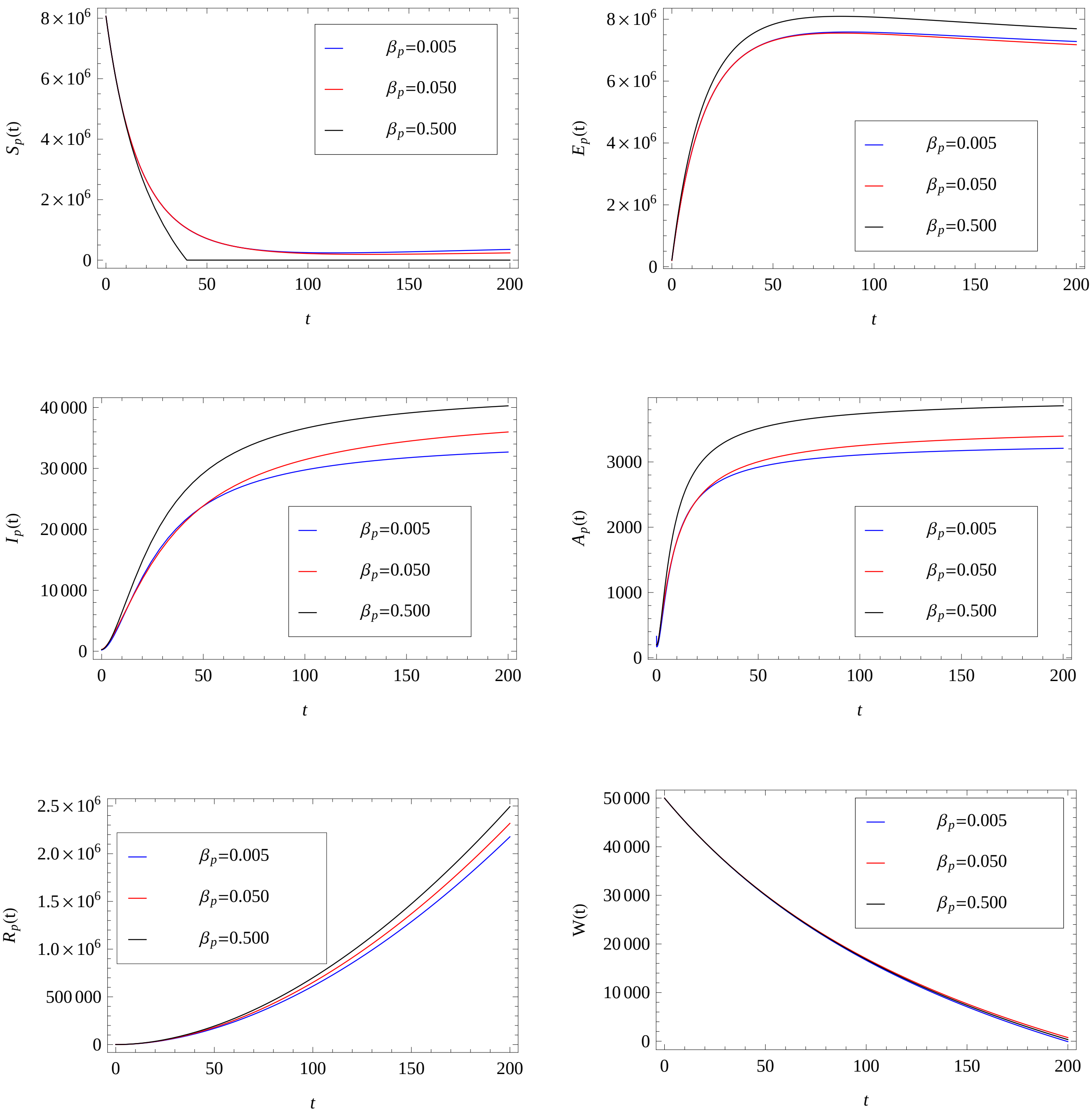

FIGURE 13. Approximated results for S_p, E_p, I_p, A_p, R_p and W for some different values of β_p in 200-days.

the model is analyzed by perturbate the mentioned parameters and numerical results are shown in figures. Further research through this area is under progress and will be reported in due time.

Acknowledgment

The authors are thankful to the white-clad angels of the medical community around the world for their unwavering assistance to Save human lives.

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